of about 10^{-7} sec. After this time the current was observed to recover (increase in magnitude) to varying degrees depending on the value of the stress. For higher stress the current showed less recovery from the breakdown.

Luminescence observations were also made during breakdown, and these observations correlate with the current pulse results in a qualitative way.[6] Previous luminescence results[7] were not applicable to the present problem since they were obtained at stresses greater than those used in this investigation. These photographs[8] (Fig. 2) showed that the breakdown occurs at discrete points in the crystal with a luminous region growing around the points. The last frame shows that the intensity of the luminescence decreases after about 200 nsec. The records demonstrate that there is good qualitative correlation between the current-time and the luminosity–time behavior, as might be expected for a dielectric breakdown.

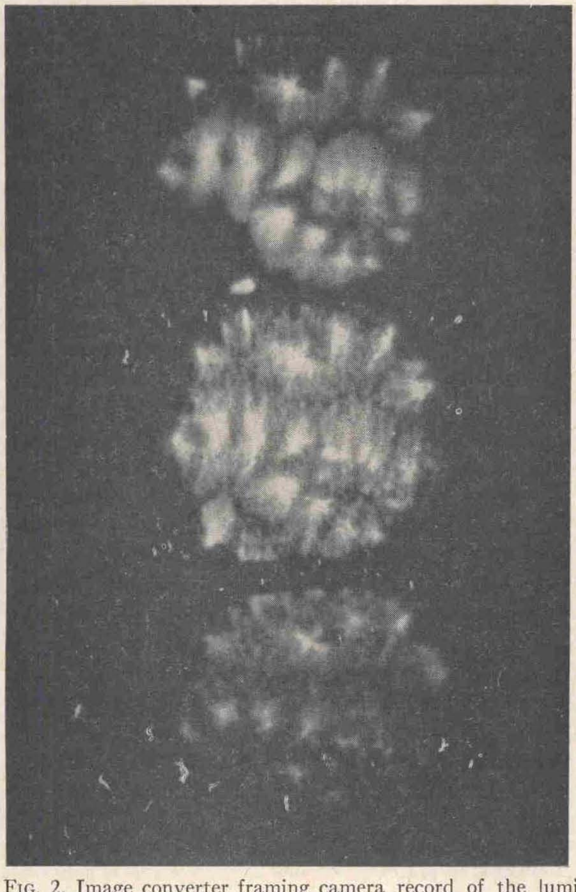

FIG. 2. Image converter framing camera record of the luminescence accompanying dielectric breakdown in $-X$ orientation disks at 19 kbar. The photographs are taken at three different times, looking down the X axis of the disk as the shock wave moves toward the observer. The first (upper) frame is taken within 50 nsec after impact and the following two frames are taken at 100 nsec intervals thereafter.

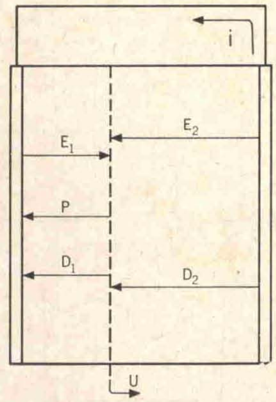

FIG. 3. Electric vectors produced in piezoelectric quartz under shock-wave loading. The position of the shock front is indicated by the dashed line. The vector arrows indicate those directions that are defined as positive in the analysis. The polarization direction shown is opposite to the propagation direction and depicts the $-X$ orientation.

Attempts to analyze the complete current–time profile on the basis of a finite constant resistivity were unsuccessful and led to the development of a mathematical model which allows for a variable resistivity under shock-wave loading.[9]

MATHEMATICAL MODEL

In Fig. 3 we consider a shock wave having a plane front and a step profile in time and space as it traverses an X-cut quartz disk of thickness l. The faces of the disk each have area A and are covered with electrodes that are connected by a short-circuit. The wave propagates from left to right with a constant velocity U in the direction of the X axis and with its front perpendicular to that axis. The region of the disk that comes under stress experiences a piezoelectric polarization P, determined by the product of the stress and the appropriate piezoelectric coefficient (the direction shown for P in Fig. 3 is indicative of the $-X$ orientation for the disk). As the wave moves through the disk, the region of polarization continually lengthens, and this causes a current i to flow in the externally connected short-circuit. The process is accompanied by fields E_1 across the stressed region and E_2 across the unstressed region, which vary with time and are oppositely directed for voltage balance.

Infinite Resistivity

We now investigate the time variations of the current i and the electric fields E_1 and E_2 mathematically. In order to provide background for interpreting the current pulses, we will first digress and review the conditions existing for infinite resistivity where break-

[6] The luminescence observations are conducted on disks in an open-circuit condition. This does not affect the qualitative comparison between the current–time measurement and the luminescence observations since at the time the shock wave enters the disk the field in the stressed region is the same under both conditions.

[7] W. P. Brooks, J. Appl. Phys. **36**, 2788 (1965).

[8] This experiment was performed by G. E. Ingram with a Space Technology Laboratories image converter camera.

[9] Analysis of the effects of shock-wave-induced finite, constant resistivity has been reported by R. H. Wittekindt, Harry Diamond Laboratories Rept. TR 922, May 1961, and Y. B. Zel'dovich, Sov. Phys.—JETP **26**, 159 (1968).

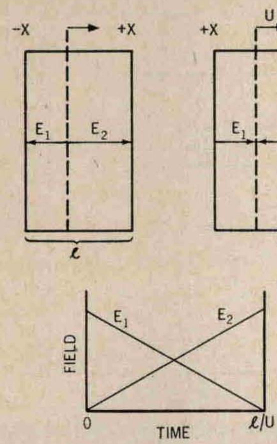

FIG. 4. Electric field diagrams for shock-loaded quartz. The upper diagrams show the directions of the electric fields E_1 and E_2 for the $+X$ (left) and $-X$ (right) orientations. The lower diagram shows the variation of E_1 and E_2 with time for the case of infinite resistivity and equal permittivities for the stressed and unstressed regions. The field E_1 in the stressed region is maximum when the shock wave enters the disk, while the field in the unstressed region E_2 is maximum at shock transit time $t=l/U$. The maximum fields are directly porportional to the magnitude of the shock stress and are equal to 5.3×10^6 V/cm at 10 kbar.

down does not occur. In Fig. 3 the shock-wave front is shown at a time t measured from the instant it entered the disk on the left, and the polarization, electric displacement, and electric field vectors are shown in directions that are defined as positive. These vectors are assumed to be uniform in magnitude over the face of the disk with directions parallel to its axis, and the subscripts 1 and 2 refer as above to the stressed and unstressed regions, respectively. Let ϵ_1 and ϵ_2 be permittivities for the two regions. We can then write the dielectric equations as

$$D_1=-\epsilon_1E_1+P \quad (1)$$

and

$$D_2=\epsilon_2E_2. \quad (2)$$

The short-circuit condition allows us to employ Kirchoff's law and write

$$E_1Ut=E_2(l-Ut) \quad \text{for} \quad 0\leq t\leq l/U. \quad (3)$$

With the assumption of infinite resistivity, it follows that the current (positive direction shown in the figure) is given by

$$i=-A(dD_2/dt), \quad (4)$$

and also that $D_1=D_2$. With the latter condition Eqs. (1)–(3) can be solved for D_2, with the result that

$$D_2=\epsilon_2PUt/[\epsilon_2Ut+\epsilon_1(l-Ut)]. \quad (5)$$

We then differentiate (5) with respect to t and substitute in (4) to get

$$i=-\alpha AUPl/[Ut+\alpha(l-Ut)]^2, \quad 0<t<l/U, \quad (6)$$

where $\alpha=\epsilon_1/\epsilon_2$. The analysis so far shows that if the resistivity is infinite and if $\alpha=1$, a constant current will result from a step-function stress imparted to the crystal.[10] The current–time waveforms observed in the $+X$ orientation and shown in Fig. 1(a) show a close correspondence to that predicted from Eq. (6).

From Eqs. (2), (3), and (5), the electric fields E_1 and E_2 resulting from the stress-induced polarization are found to be given by

$$E_1=P(l-Ut)/[\epsilon_2Ut+\epsilon_1(l-Ut)],$$

$$E_2=PUt/[\epsilon_2Ut+\epsilon_1(l-Ut)]. \quad (7)$$

Thus, as shown in Fig. 4, the field in the stressed region of the disk is maximum at the time the shock wave enters the disk, and then decreases linearly (for $\alpha=1$) in time to a value of zero at wave transit time, which is of the order of 1 μsec. On the other hand, the field in the unstressed region is zero when the shock wave enters the crystal and increases to a maximum value at wave transit time. When the direction of propagation of the shock wave is reversed relative to the polarity of the X axis, the field directions are reversed.

The distinctly different field–time environment in the stressed and unstressed regions of the disk permits identification of the region through which the breakdown occurs. Breakdown is observed to occur early in time, which corresponds to the time of maximum field in the stressed region. Hence, breakdown occurs through the stressed region.

Finite Resistivity

We now attempt to obtain a quantitative interpretation of recovery from breakdown observed in the $-X$ orientation experiments by developing a mathematical model which includes the effect of a variable resistivity. First, a very general assumption is made that some process occurs uniformly throughout the stressed region 1 in such a way as to alter the rate at which D_1 changes as a function of time. We express this effect by introducing a loss factor F in the equation

$$dD_2/dt=(dD_1/dt)-F(t), \quad (8)$$

which now, in general, relates D_1 and D_2 in a way other

[10] If $\alpha<1$ the current value at $t=0$ is greater than the value at $t=l/U$ while for $\alpha>1$ the opposite is true. However, as can be seen by solving Eq. (6) for

$$\int_0^{l/U} i(t)\,dt,$$

the charge Q that passes through the short circuit from $t=0$ to $t=l/U$ is equal to PA, independent of the value of α for this case of infinite resistivity. Thus, in an experimental result, the deviation of the charge Q from the value PA is a measure of the extent of the breakdown or of the average finite resistivity of the disk during wave transit time.